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Sensitivity of channel network planform laws and the question of topologic randomness

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Abstract. The random topology model approximately predicts all orientation-free empirical laws of channel network planform. Explanation of the model's success has crucial significance for research direction. *Shreve* [1975] proposed the explanation that random factors dominate network development. An alternative explanation is that these geomorphologic laws are insensitive to topologic variability. *Kirchner* [1993] tested the sensitivity of the Horton ratios to particular types of topologic distribution. We extend the work of Kirchner to all orientation-free empirical laws of channel network planform. We test the sensitivity of the topologic analog of each law using non–topologically random network test samples. One type of sample is obtained with the stochastic variable-parameter "Q model" of network growth, used as a test example, where the probability of tributary development on interior and exterior links is allowed to vary. We show that the coefficients of all channel network planform laws vary with model parameter Q. It is possible that these laws are sensitive also to other models of network growth. We conclude that the success of the random model in approximately predicting geomorphologic laws may not be due to the insensitivity of these laws and is a result that remains unexplained.

1. Introduction

Landscape evolution by fluvial erosion is among the earth sciences phenomena that usually occur over time periods too long for observation, and study of the physical processes involved relies strongly on inference from landscape form [*Gilbert*, 1877]. The most striking morphologic feature of fluvially eroded landscapes is the land-surface tiling by valleys nested within larger valleys, their bottoms forming a connected network with the appearance of a bifurcating arborization. Through the valley network extend the stream channels that carry flow and sediment from the landscape. That valley connectivity and continuity of slope show such "nice adjustment" that they appear designed to accommodate the network of channels testifies that the valley is "the work of the stream which flows in it" [*Playfair*, 1802, p. 102].

It is not surprising that early geomorphologists judged the configuration of channel networks as most worthy of study, even as "the key to landscape" [*Zernitz*, 1932, p. 68], and searched for patterns and regularities in such networks. Channel network morphology may reflect regional tectonics, local geologic structure, prevailing erosional mechanisms, and climate [see *Kirchner*, 1993, p. 591]; in turn it affects hydrologic processes and fluxes [e.g., *Taylor and Schwarz*, 1952; *Surkan*, 1969; *Kirkby*, 1976; *Rodríguez-Iturbe and Valdés*, 1979; *Troutman and Karlinger*, 1986]. With the current availability of digital elevation maps and satellite imagery, from which approximate representations of channel networks may be obtained, the ability to interpret network morphology could be useful for the inference of geophysical processes and geologic properties at the planetary scale.

Early descriptions of channel network morphology were mainly qualitative [e.g., *Davis*, 1909; *Zernitz*, 1932]. Quantitative analyses based on centrifugal branch-ordering systems (*Gravelius* [1914], cited by *Jarvis and Woldenberg* [1984]), borrowed from the life sciences, were unsuccessful at identifying regularities owing to lack of correlation of centrifugal order with channel size. *Horton* [1932] revived quantitative analysis with the introduction of a centripetal branch-ordering system. Centripetal ordering, using rules by *Strahler* [1952] (Figure 1a), remains in current use in fluvial geomorphology and was adopted in studies of organic networks [*Jarvis and Woldenberg*, 1984, pp. 9–10, 101]. For a detailed review of the history of branch-ordering systems, see *Woldenberg* [1997]. A sequence of channel reaches of the same centripetal order is called a stream. The concepts of drainage density and stream frequency, defined as the average length of channels and average number of streams per unit terrain area, respectively, were also major contributions by *Horton* [1932].

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Channel Network Planform Laws

Horton [1945] found that the decrease in number and increase in mean length of streams with centripetal order is approximately geometric and hypothesized that the increase in mean subbasin area is also geometric, as confirmed by *Schumm* [1956]. These statistical relations are known as "Horton's laws" of stream numbers (Figure 1b), lengths, and areas. Their respective series ratios are designated the bifurcation, length, and area ratios (R_B, R_L, and R_A). *Horton* [1945, p. 295] proposed that R_B and R_L, together with basin area (A), basin order (order of the outlet stream, Ω), and the average length of first-order channels ($\bar{L}_1$), "determine completely the composition of a stream system" and permit estimation of drainage density (D) and stream frequency (F_s). *Horton* [1945, p. 291] interpreted these three laws as revealing that "[Playfair's] nice adjustment goes far beyond the matter of declivities," reflecting organized patterns of channel connectivity, or network topology.

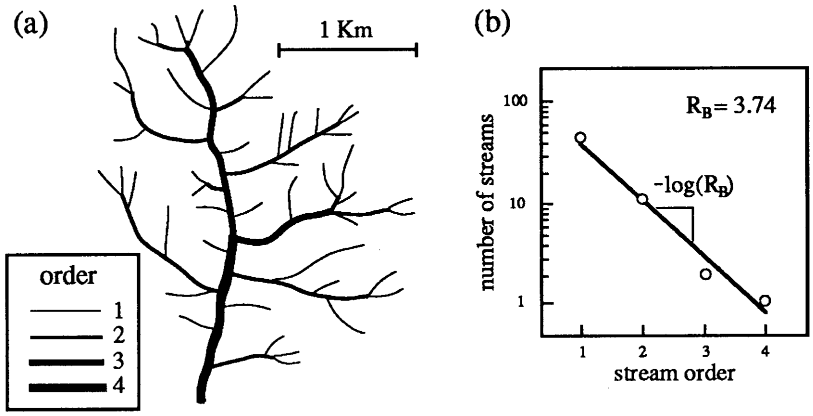

Figure 1. (a) Illustration of Strahler stream ordering rules: Source channels are streams of order 1, and where two streams of the same order, ω, join, the receiving stream has order $\omega + 1$. The fourth-order channel network pictured discharges into a fifth-order tributary stream of Old Man Creek, Iowa. The number of streams of each order ω, N_ω, are $N_1 = 46$, $N_2 = 11$, $N_3 = 2$, and $N_4 = 1$. The number of source streams, N_1, is designated the magnitude of the network, μ. (b) Horton diagram of stream numbers for the network in Figure 1a: log (N_ω) is plotted against ω. The bifurcation ratio is $R_B = 3.74$, obtained from the slope of the regression line, $(-\log(R_B))$. The geometric-mean ratio is $G_B = 3.58$, obtained from the slope of the line connecting the first and last points in the diagram, given by (4a), for $\mu = 46$ and $\Omega = 4$.

Much effort was subsequently expended to find other geomorphologic laws resulting from this perceived organization of topologic patterns. A number of empirical statistical laws were established, including laws of channel network planform by *Langbein et al.* [1947], *Hack* [1957], *Melton* [1958], and *Gray* [1961], among others, relating quantities such as basin area, mainstream length, total length of channels, and others. These laws constitute some of properties 1–7, listed below.

The Random Topology Model

Shreve [1966] and subsequent researchers dispelled the notion that the above geomorphologic laws imply topologic orderliness or distinctive characteristics. This had been suggested earlier by *Leopold and Langbein* [1962], who demonstrated that networks generated by random walks on a square lattice were in accord with Horton's laws, leading *Milton* [1966] to dismiss these laws as "irrelevant" to geomorphology.

Shreve [1966] investigated Horton's law of stream numbers with the intent of explaining the law's generality and insensitivity to environmental factors (first noted by *Horton* [1945, p. 303]). This law is purely topological, that is, determined by the channel connectivity pattern, and to study it, *Shreve* [1966] introduced the concept of topologically distinct channel networks (TDCN) with a given number of channel sources, or "network magnitude" (Figure 2). Shreve found that Horton's law of stream numbers holds approximately for all TDCN that can be constructed with any given magnitude; hence it is "inherent in the definition of stream order" [*Shreve*, 1966, p. 30]. *Shreve* [1966] further showed that most TDCN that can be constructed have R_B values in the range typically observed in nature, with a modal value of about 4.

Having disproved that Horton's law of stream numbers and typical R_B values require particular topologic patterns, Shreve then proposed the diametrically opposite hypothesis: that "in the absence of geologic controls a natural population of channel networks will be topologically random"; that is, "all topologically distinct networks [TDCN] with given number of links are equally likely" [*Shreve*, 1966, p. 27]. Shreve's definition of network randomness concerns topology alone and is descriptive rather than causative; that is, it assumes nothing regarding network growth processes.

All geomorphologic laws other than Horton's law of stream numbers involve metric quantities, whose variability has both topologic and metric components. For example, variability in mainstream length is due partly to variability in the number of links comprising the mainstream (its "topologic length") and partly to variability in individual link lengths. Similarly, basin area is determined by the number of links and by the size of areas contributing flow to individual links. An approximately linear relation may be expected between each metric variable and its topologic component, especially for large numbers of links, provided that no significant correlations exist between link length or contributing area and topologic variables [*Shreve*, 1974]. For example, if mean link length increased with stream order, mainstream length would not vary linearly with topologic length.

Link length distribution has been found to vary with stream order and other topologic variables in some river basins [e.g., *Smart*, 1968; *Shreve*, 1969; *James and Krumbein*, 1969; *Smart*,

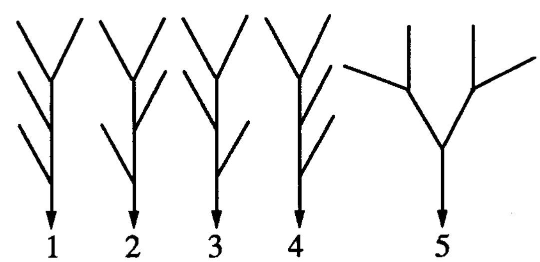

Figure 2. The five topologically distinct channel networks (TDCN) that can be constructed having four channel sources (magnitude 4). Each network is represented by a binary rooted tree graph [*Melton*, 1959], that is, a line graph drawn on a surface where no more than three segments, or "links," join at a point and that has a specified "root," or outlet location (here indicated by an arrowhead) "Nodes" are points of link confluence. "Exterior links" represent source channels, and "interior links" connect two nodes.

1969, 1972, 1978, 1981; *Ghosh and Scheidegger*, 1970; *Krumbein and Shreve*, 1970; *Mock*, 1971; *Abrahams and Campbell*, 1976; *Montgomery and Dietrich*, 1989], but this variation is unlikely to affect significantly the approximately linear relation between a metric variable and its topologic component for large numbers of links [*Kirchner*, 1993, p. 593]. Link length and area distributions (sampled by, e.g., *Strahler* [1954], *Schumm* [1956], and *Maxwell* [1960]) have only slight influence on linearity [*Shreve*, 1974]. Using the hypothesis of linearity, *Smart* [1968] obtained good correlations between observed stream lengths and topologic stream lengths predicted by the random topology model.

Under the assumption of linearity, we obtain topologic analogs of channel network planform laws by replacing each metric variable with its topologic component [*Shreve*, 1967]. Predictions of the random topology model for the topologic-analog laws, which in some cases were coupled with observed distributions of link lengths and areas, were shown to approximate the respective geomorphologic law, as summarized by *Shreve* [1975, p. 527] (edited):

> Without any adjustable parameters and with no other input than the observed distributions of link lengths and associated areas, [the random topology model] has successfully predicted quantitatively the following observed [properties]:
>
> [property 1] the distributions and numerical values of R_B (*Shreve*, 1966), R_L (*Smart*, 1968; *Shreve*, 1967, 1969), and R_A (*Shreve*, 1969)
>
> [property 2] the correlations between R_B and R_L and between stream numbers and stream lengths (*Smart*, 1968)
>
> [property 3] the statistical distributions of second-order stream lengths, Schumm lengths, and areas (*Shreve*, 1969)
>
> [property 4] *Melton*'s (1958) proportional relationship between stream frequency and the square of the drainage density (*Shreve*, 1967)
>
> [property 5] *Hack*'s (1957) variation of mainstream length with basin area (e.g., *Shreve*, 1974)
>
> [property 6] *Langbein et al.*'s (1947) relationship between distance-weighted area and basin area (*Werner and Smart*, 1973)
>
> [property 7] *Gray*'s (1961) relationship between distance from outlet to "centroid" and mainstream length (*Werner and Smart*, 1973).

While violations of some of these laws, or properties, have been documented for individual networks, they approximate the aggregate central tendency of published data [*Abrahams*, 1984]. As noted by *Shreve* [1975, p. 529], even though stated in deterministic terms, the above laws are all probabilistic in essence, and "indicate only general tendencies in large populations rather than exact relationships in individual cases."

Search for an Explanation

Explanation of the random topology (RT) model's approximate prediction of properties 1–7, above, remains far from established and subject to debate [e.g., *Watson*, 1966, 1969; *Thakur and Scheidegger*, 1968; *Krumbein and Shreve*, 1970; *Howard*, 1972; *Smart*, 1973; *Shreve*, 1974, 1975; *Schumm*, 1977; *Jarvis and Sham*, 1981; *Abrahams*, 1984, 1987; *Mesa and Gupta*, 1987; *Kirchner*, 1993]. What explains the RT model's predictive ability?

Shreve [1975, p. 529] proposed two inferences, or explanations: (1) the topologic component of geomorphologic laws is dominant, and (2) random factors dominate network development. Inference 1 is almost certainly correct, as agreement between metric and topologic-analog laws could hardly be coincidental (and supports the above assumption of linearity). Is explanation 2 valid?

This question is crucial for the direction of future research and for the best usage of the RT model. If random factors dominate network development, then it is those empirical properties of natural networks that disagree with RT model predictions that may be geomorphically significant and merit our attention. The RT model may be useful in isolating those properties. Also, why the physical processes involved in network development might have dominantly random effects on network topology becomes a fundamental research question in geomorphology [*Kirkby*, 1976, p. 197].

The question of the validity of inference 2 has been approached in three different ways. One approach has been to oppose inference 2 on the argument that physical processes are intrinsically deterministic [e.g., *Watson*, 1966; *Howard*, 1972; *Schumm*, 1977], or to defend inference 2 on the argument that physical processes are intrinsically random [e.g., *Leopold and Langbein*, 1963; *Langbein*, 1964; *Krauskopf*, 1968; *Mann*, 1970], or that process complexity and sensitivity to initial and boundary conditions preclude deterministic modeling [e.g., *Thakur and Scheidegger*, 1968; *Krumbein and Shreve*, 1970; *Smart*, 1973; *Shreve*, 1975]. *Smart* [1979] classified these arguments as philosophical, for they cannot be tested by current scientific methods. These arguments address the legitimacy of the RT model rather than the quality of its predictions [*Shreve*, 1979, p. 170]. We note that if simple deterministic processes of channel development were influenced by physical factors not correlated with any topologic variables, such processes would appear as random in the topologic domain. Hence topologic randomness does not preclude deterministic processes, nor does it imply complexity.

A second approach to the question of validity of inference 2 has been to devise progressively discriminating tests of the RT model, as argued for by *Kirkby* [1976]. The RT model has been tested often against collected data and has rarely been rejected. Direct testing of TDCN frequencies is feasible only for small networks, with six or fewer source channels [e.g., *Werner and Smart*, 1973], because the number of TDCN, given by the formula by *Cayley* [1859] [*Shreve*, 1966, equation 13], increases rapidly with network magnitude. The number of TDCN with magnitude 7, 10, and 50, is 132, 4862, and about 2.52×10^{26}, respectively. For this reason, tests of the RT model have used topologic variables which individually have limited informational content [e.g., *Jarvis and Werritty*, 1975; *Abrahams and Mark*, 1986; *Uylings et al.*, 1989; *Verwer et al.*, 1992]. However, the combined informational content of all topologic variables predicted by the RT model has not been evaluated. *Abrahams and Mark* [1986] argued that the small significance level of 5% used in most of these tests for rejection of the RT model has resulted in favorable bias for the model.

Despite their limitations, statistical tests have successfully identified systematic deviations in nature from RT model predictions. These deviations were reviewed by *Abrahams* [1984, 1987]. *Abrahams* [1987, p. 164] summarized evidence that each systematic deviation from the RT model results from either steep or low terrain slope or from space filling and geometric properties of channel networks. The latter affect, in particular, the arrangement of tributaries along a mainstream. For example, the number of two consecutive tributaries that join the mainstream from opposite sides (right and left) is found usually to be higher than the number joining it from the same side, especially when one or both tributaries have large basin areas [*James and Krumbein*, 1969; *Flint*, 1980; *Abrahams*, 1984]. The RT model predicts that these occurrences will be equally frequent. Attempts at simulating and quantifying the topologic effects of space filling by simulation include those of *Karlinger and Troutman* [1989], *Goodchild and Mark* [1985], *Goodchild* [1988], and *Goodchild and Klinkenberg* [1993].

A third approach to the question of validity of inference 2 has been to question the sensitivity of channel network planform laws to topologic variability. RT model deviations due to space filling or other possible factors are not detected by any of the above planform laws (properties 1–7), and *Abrahams* [1987, p. 156] attributes this failure partly to insensitivity of these laws. One source of insensitivity is the aggregation of topologic data from networks with diverse physical conditions, such as slope and orientation, resulting in the "canceling out" of opposing properties [e.g., *Werritty*, 1972; *Jarvis and Werritty*, 1975; *Flint*, 1980; *Abrahams and Mark*, 1986]. *Kirchner* [1993] raised the important question of insensitivity of the Horton ratios to network samples having opposing topologic properties, in the absence of aggregation.

The Question of Sensitivity

Kirchner [1993] investigated the sensitivity of the Horton ratios for non–topologically random samples of networks. Kirchner created non–topologically random samples by first generating an original topologically random sample of networks and then selecting from that original sample those networks having specified topologic characteristics. Kirchner selected networks having one given topologic variable (such as maximum topologic length, or "diameter") larger than, or smaller than, the median value of the original sample. Kirchner demonstrated that for such non–topologically random samples, the distributions of the Horton ratios deviate little from those in the original topologically random sample.

Kirchner concluded that the RT model's prediction of the typical values of the Horton ratios does not constitute evidence in favor of the RT model because similar values are observed also in many non–topologically random samples. *Kirchner*'s [1993] results reinforce the perception of the Horton ratios, and hypothetically some geomorphologic laws, as being insensitive to TDCN distribution.

A different method for creating non–topologically random test samples is to generate each network using a stochastic topologic model of network growth. In geomorphology, examples include the convergent-growth (merging of channels formed independently) model of *Leopold and Langbein* [1962] and the headward-growth (addition of tributaries to preexisting channels) models of *Howard* [1971] and *Dacey and Krumbein* [1976], among others.

The models of *Dacey and Krumbein* [1976], developed for channel networks, were generalized into the variable-parameter "*Q* model" by *Van Pelt and Verwer* [1985] in the field of neuroanatomy. The *Q* model (section 2) assigns a variable probability to development of tributaries in interior and exterior links (defined in Figure 2). By testing against simulation results, *Dacey and Krumbein* [1976] rejected the hypothesis that channel networks grow principally by tributary development on exterior links (source channels). This growth mechanism, however, predicts well the topologic properties of dendrites of nonpyramidal neurons [*Van Pelt et al.*, 1992].

Contrary to channel networks, the topologic properties of neuronal dendrites differ widely from RT model predictions for the "ambilateral classes" of networks, that is, classes of TDCN which cannot be distinguished topologically from one another in three-dimensional space [*Verwer and Van Pelt*, 1983]. This is also the case for various other organic networks. Growth models that predict the topologic properties of particular organic networks include those of *Harding* [1971], *Berry et al*. [1975], *Hollingworth and Berry* [1975], and *Van Pelt and Verwer* [1985, 1986]. *Van Pelt et al*. [1992] found that two types of neuronal dendrite have topologic properties well represented by best-fit *Q* parameter values.

These results demonstrate that topologic variables are sensitive to some types of topologic distribution, as suggested by *Troutman and Karlinger* [1994], and provoke the question of whether the channel network planform laws (properties 1–7, above) are sensitive to the TDCN frequencies produced by these growth models.

Objective

The objective of this paper is to test the sensitivity of the channel network planform laws above (properties 1–7) to nonuniform TDCN distributions.

We generate non–topologically random test network samples and compute the parameters of the topologic analogs of properties 1–7 for each sample. Two types of sample are tested. A sample of type 1 is obtained by selection of networks with specified topologic properties from an original topologically random sample. This is the method introduced by *Kirchner* [1993]. A sample of type 2 is obtained by generating each network using a variable-parameter stochastic topologic model of network growth. The "*Q* model" is used as a test example.

The *Q* model is described in section 2. The methods used to generate each test sample are described in section 3. The coefficients of the topologic analogs of properties 1–7 are computed for each test sample in section 4, and conclusions are presented in section 5. The approximate size of a subsample required to distinguish statistically between topologically random and non–topologically random test samples is not computed. To distinguish between samples, we rely instead on the visual representation of the topologic analogs of properties 1–7 and on the computed coefficients and standard deviations. Sample sizes required to distinguish between samples based on any of these properties will depend on the magnitudes of the networks sampled.

The present analysis is strictly topological, giving no consideration to link lengths, junction angles, or any metric variables. Results and conclusions could be different for networks embedded in space, a topic of current research by the authors.

2. The *Q* Model of Network Growth

The *Q* model of network growth [*Van Pelt and Verwer*, 1985] was introduced in the field of neuroanatomy to represent possible growth mechanisms of neuronal dendrites of various types. The usefulness of such growth models for inference of natural growth mechanisms was argued for by *MacDonald* [1984]. The *Q* model is used in this paper as a test example only.

Model Description

The *Q* model represents network growth by sequential branching of links, that is, by appending one new tributary link to one preexisting link at each growth step. A magnitude μ network is obtained after $\mu - 1$ growth steps. The choice of which link will branch (i.e., develop a new tributary) at each growth step is made stochastically. The probability of a given link branching is p_i if the link is interior and p_e if the link is exterior. Model parameter Q relates p_i and p_e and takes values in [0, 1]

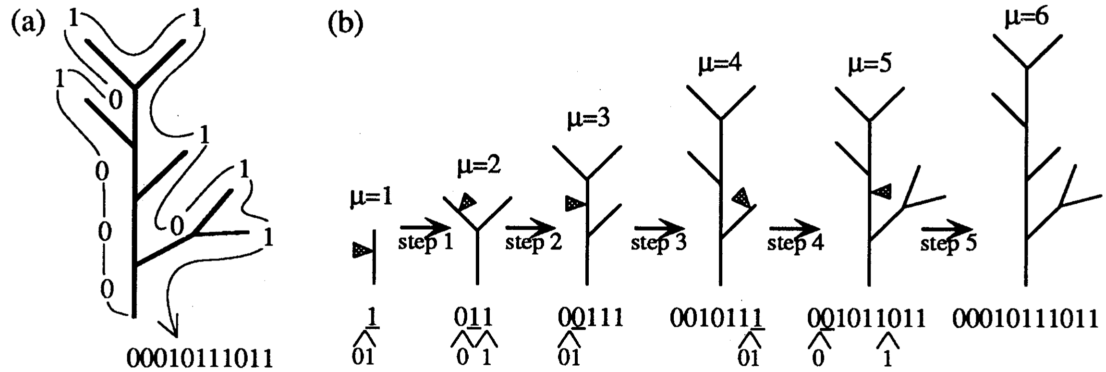

Figure 3. (a) Representation of a network's topology by a binary string, following Lukasiewicz's convention: Interior and exterior links are represented by "0" and "1," respectively, and are listed in sequence starting with the root link and moving from left to right around the network. (b) Example sequence of five growth steps yielding the magnitude 6 ($\mu = 6$) network in Figure 3a. The link chosen for branching, and the side (left or right) to which the new tributary link will be appended, is indicated by an arrowhead. The digit representing the link chosen is underlined in the binary string at the bottom, and the two new digits to be inserted are indicated.

$$Q = \frac{p_i}{p_i + p_e} \tag{1}$$

Because one branching event must occur at each growth step, the sum of branching probabilities of all links in the growing network must equal unity. A network with μ exterior links (a magnitude μ network) has $\mu - 1$ interior links, and we have

$$\mu p_e + (\mu - 1)p_i = 1 \tag{2}$$

Combining (1) and (2) yields

$$p_i = \frac{Q}{\mu - Q} \tag{3a}$$

$$p_e = \frac{1 - Q}{\mu - Q} \tag{3b}$$

For $Q = 1/2$ (i.e., $p_i = p_e$), any link (or "segment"), be it interior or exterior, is equally likely to branch at each growth step, and the process is called "segmental growth." If when an interior link branches the probability of appending the new tributary link to the right- or left-hand side of that link is the same, then segmental growth yields any TDCN of given magnitude with the same probability and is a causative model of topologic randomness [*Dacey and Krumbein*, 1976, p. 157].

For $Q = 0$ (i.e., $p_i = 0$), only exterior (or "terminal") links branch, and the process is called "terminal growth." For $Q = 1$ (i.e., $p_i = 1$), only interior links branch, and this process yields exclusively order 2, that is, "fishbone-shaped," networks. Any given TDCN can be produced using any value of Q (except for $Q = 1$); however, its probability of occurrence depends on Q. Lower Q values are more likely than higher Q values to produce high-order networks.

Models of either segmental or terminal growth have been used, for example, by *Howard* [1971], *Smit et al.* [1972], *Berry et al.* [1975], *Berry and Bradley* [1976], and *Dacey and Krumbein* [1976]. The latter authors also used a growth model where $p_i = 2p_e$, and the Q model is a generalization of their work to any p_i/p_e ratio.

The mean values of various topologic variables vary continuously with parameter Q [e.g., *Van Pelt and Verwer*, 1985, 1986]. Comparing expected topologic measures with observed data, maximal-likelihood Q values can be determined, and some Q values may be ruled out at some confidence level. For example, *Van Pelt et al.* [1992] found the best-fit values $Q = 0.11$ for Purkinje neuronal dendrites, and $Q = 0$ for nonpyramidal dendrites. Inference of most-likely growth parameters from network topology for networks embedded in space may, however, require consideration of spatial constraints, which have so far been neglected in such inference methods.

Computer Code

A computer code, designated Q.f, was written in Fortran to simulate network growth by the Q model. Networks were represented as binary strings, following Lukasiewicz's convention [*Berge*, 1958] (Figure 3a).

Growth of a network of magnitude μ starts with one initial link and requires a number $\mu - 1$ of branching events, or growth steps. Choice of which link will branch at each growth step is done by comparison of a random variable with the link branching probabilities, p_i (3a) for any interior link, and p_e (3b) for any exterior link. Once a link is chosen for branching, the new link is appended to the left- or right-hand side of the branching link with equal probability, as determined by a random variable.

Figure 3b shows an example of five growth steps, or branching events. A branching event results in the addition of two new links: a new exterior link (appended to either the left- or right-hand side of the branching link) and a new interior link that results from the partition of the branching link into two. Following each branching event, two digits representing these new links are added to the binary string that represents the network. If the new link is appended to the left-hand side of the branching link (as in steps 1, 3, and 4 in Figure 3b), the digits "01" are added preceding the digit representing the branching link. If the new link is appended to the right-hand side of the branching link (as in steps 2 and 5 in Figure 3b), the digit "0" is added preceding the digit representing the branching link, and the digit "1" is added at the end of the sequence of digits (the "interval sequence") that represents the subnetwork defined by the branching link. To find the end of the interval sequence, we start at its first digit (the digit represent-

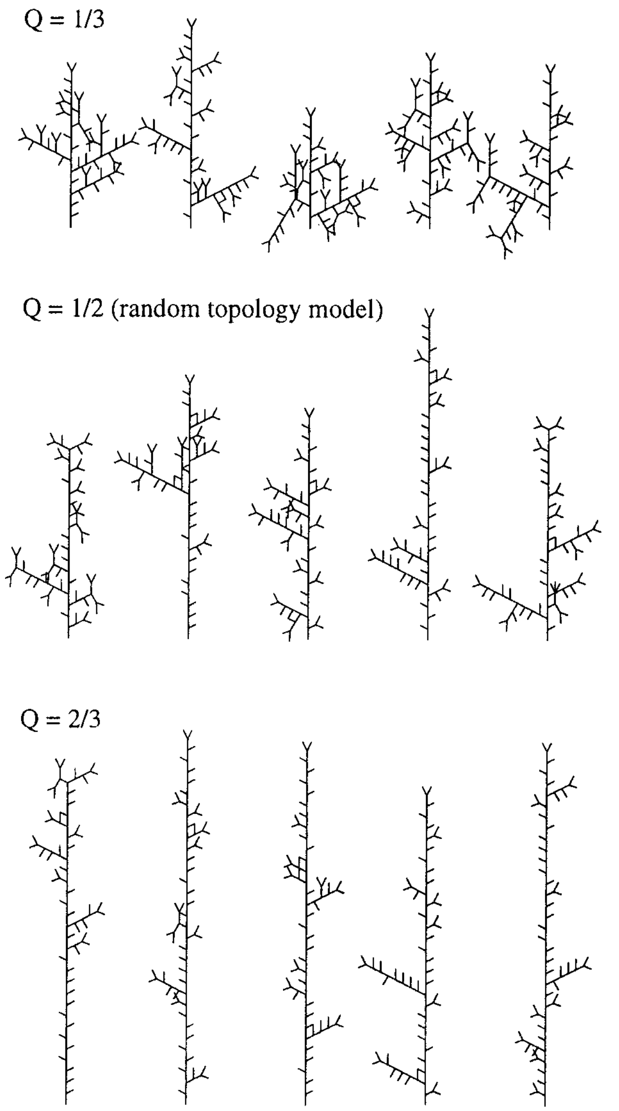

Figure 4. Five magnitude-50 networks drawn at random from test sample G (created with $Q = 1/3$), test sample 0 ($Q = 1/2$), and test sample H ($Q = 2/3$). Five networks from each test sample suffice to illustrate network characteristics favored by different Q values.

ing the branching link) and move forward in the string counting the number of digits (links), n_t, and the number of "1" (exterior links), n_e. Because a network with n_e exterior links has a total of $2n_e - 1$ links, the end of the interval sequence is found when n_t and n_e satisfy the equality $n_t = 2n_e - 1$. If the branching link is exterior, the interval sequence is the single digit "1" (as in step 2 of Figure 3b). In step 5 of Figure 3b the end of the interval sequence "01011" is found when $n_t = 5$ and $n_e = 3$.

Computer code Q.f is available from the authors upon request.

3. Test Network Samples

We use two sets of test network samples: 10 samples of networks of magnitude 50, including a topologically random sample designated 0 and 9 non–topologically random samples designated "A" through "I"; and 10 samples of networks of mixed magnitudes, including a topologically random sample designated "0 mix" and 9 non–topologically random samples designated "A mix" through "I mix." Each sample contains 5000 networks.

Samples of Magnitude-50 Networks

Sample 0 was constructed using the Q model computer code (section 2), with $Q = 1/2$, for which all magnitude 50 TDCN are equally likely to be created. This sample is therefore drawn, with replacement, from a topologically random population and is here said to be a "topologically random sample." A more efficient algorithm for drawing networks from a topologically random population is given by *Shreve* [1974, p. 1172].

Non–topologically random test samples are of type 1 (samples A through D) and type 2 (samples E through I). Test samples of type 1 were obtained by selection of networks having specified topologic properties from an original topologically random sample, a method introduced by *Kirchner* [1993]. Test samples of type 2 were obtained using the Q model computer code.

Test samples of type 1 are selected subsets of size 5000 of an original topologically random sample of size 10,000, which was obtained in the same manner as sample 0. Each of samples A through D contains all networks in the original random sample whose value of one specified "selection variable" is greater than or smaller than the median of the original sample. To make up 5000 networks it is necessary in some cases to include some of those networks with values equal to the median.

Sample A contains 5000 networks whose R_B is smaller than or equal to the median value, $\tilde{R}_B$, in the original random sample. Samples B through D contain networks whose diameter (d), topologic width (w), and number of tributary-source links (nTS), respectively, are greater than or equal to the median values of these variables in the original random sample. Variables d, w, and nTS are defined next.

The diameter, d, is the maximal number of links positioned in linear sequence from the outlet to a channel source (the longest topologic path). The topologic width, w, is the maximal number of links having the same topologic distance (path length) to the network outlet. The value nTS is the number of tributary-source links in the network, defined as source links (i.e., exterior links) that are tributary to a link of magnitude 3 or higher.

Test samples of type 2 were constructed using the Q model computer code (section 2). Samples E through I were constructed with Q values of 0, 1/6, 1/3, 2/3, and 5/6, respectively. Samples E through I and sample 0 have Q values at intervals of 1/6. For $Q < 1/2$ we have, from (1), $p_e > p_i$; that is, tributary development on any one exterior link is more likely than on any one interior link. For example, in sample G, we have $Q = 1/3$ and, from (1), $p_e = 2p_i$. The resulting distribution of TDCNs favors higher Strahler orders than in a topologically random sample. For $Q > 1/2$ we have, from (1), $p_i > p_e$. Sample H was constructed with $Q = 2/3$, and from (1), we have $p_i = 2p_e$. Values of Q larger than 1/2 favor more elongate networks, of lower Strahler orders (Figure 4).

Samples of Mixed-Magnitude Networks

Study of the topologic analogs of geomorphologic laws that involve basin area requires a range of network magnitudes. Each mixed-magnitude test sample contains 10 networks of each magnitude in [1, 500]. These samples were created by methods analogous to those for the magnitude 50 samples 0 and A through I (above), and are designated "0 mix" and "A mix" through "I mix," respectively. Table 1 summarizes the method by which each test sample was created.

4. Topologic Analogs of Channel Network Planform Laws

In this section we compute the parameters of the topologic analog of each geomorphologic law listed in section 1 (properties 1–7) for the test network samples described in section 3. Variable notation is summarized in the notation section.

A computer code, designated topovars.f, was written in Fortran to compute the topologic variables from the binary strings representing networks. Computational algorithms were developed by the authors and are not described here because of space limitations. The computer code is available from the authors upon request.

Property 1: Horton's Laws and Ratios

Shreve [1966, p. 30] showed that "no [TDCN] can depart indefinitely far from Horton's [law of stream numbers]," and in this sense this law is "inherent in the definition of stream order." Hence this law may not be attributed "to either orderly evolution or random development" [*Bowden and Wallis*, 1964, p. 769]. Most TDCN also comply approximately with the topologic analogs of Horton's laws of stream lengths and areas [*Shreve*, 1969].

The generality of Horton's laws among TDCN is most easily explained for the law of stream numbers, as follows. The first and last points in a Horton diagram of stream numbers (Figure 1b) are determined by network magnitude and order ($N_1 = \mu$ and $N_\Omega = 1$), while the intermediate points (N_ω, $1 < \omega < \Omega$) have variable positions. Shreve showed that the domain of variation for these points is narrow, because "for every stream of given order, except the first, there must be at least two streams of the next lower order" [*Shreve*, 1966, p. 30]. This range is a narrow parallelogram whose diagonal represents a geometric series with ratio G_B (the geometric-mean bifurcation ratio) determined by μ and Ω (4a) [*Shreve*, 1969, p. 21]. R_B, obtained by regression, approximates G_B (4b):

$$G_B = \mu^{1/(\Omega-1)} \tag{4a}$$

$$R_B \approx G_B \tag{4b}$$

Given any set of stream numbers, we can arrange those streams to form a finite number of TDCN. *Shreve* [1966] demonstrated that those sets of stream numbers which approximate a geometric series with ratio 4.0 allow the largest number of TDCN. Therefore, in a topologically random population of magnitude-μ networks, the most frequent, or modal order, $\Omega_M(\mu)$, is that for which G_B is closest to 4.0 [*Shreve*, 1966]. From (4)

$$\Omega_M(\mu) = \text{round}\left(\frac{\log \mu}{\log 4} + 1\right) \tag{5}$$

where "round( )" denotes the closest integer.

The topologic analogs of Horton's laws of stream lengths and areas substitute the number of links in a stream (the stream's "topologic length") and in a network (the network's topologic "Schumm length") for stream length and basin area, respectively. The topologic-analog length and area ratios are here indicated by R_L^t and R_A^t, and the geometric-mean ratios are G_L^t and G_A^t. For a network of order Ω, G_A^t depends on the total number of links, equal to $2\mu - 1$ (6a), and G_L^t depends on the topologic length of the highest order stream, L_Ω^t (7a). Ratios R_L^t and R_A^t approximate G_L^t and G_A^t, respectively (6b and 7b):

$$G_A^t = (2\mu - 1)^{1/(\Omega-1)} \tag{6a}$$

$$R_A^t \approx G_A^t \tag{6b}$$

$$G_L^t = L_\Omega^{t\,1/(\Omega-1)} \tag{7a}$$

$$R_L^t \approx G_L^t \tag{7b}$$

Smart [1968, equation (13a)] derived an expression for the average topologic length of streams of order ω, $\bar{L}_\omega^t$, as a function of the set of stream numbers, $\{N_i,\ 1 \le i \le \omega\}$, under the assumption of topological randomness:

$$\bar{L}_\omega^t = \prod_{i=2}^{\omega} \frac{N_{i-1} - 1}{2N_i - 1} \tag{8}$$

For the RT model the ratio N_{i-1}/N_i tends to 4 at large order i, and (8) approximates Horton's law of stream lengths with $R_L^t = 2$ [*Smart*, 1968].

Approximating the set of stream numbers by a geometric series with the ratio given by (4a), we obtain, using (8), the mean topologic length of the outlet stream of order-Ω networks, $\bar{L}_\Omega^t$, for the RT model:

$$\bar{L}_\Omega^t = \prod_{\omega=2}^{\Omega} \frac{\mu^{1-(\omega-2)/(\Omega-1)} - 1}{2\mu^{1-(\omega-1)/(\Omega-1)} - 1} \tag{9}$$

G_L^t for topologically random samples is approximated by substituting $\bar{L}_\Omega^t$ in (9) for L_Ω^t in (7a).

The quality of approximations (4b), (6b), and (7b) can be appreciated from Figure 5. Deviations from (7b) are more pronounced, especially for orders other than $\Omega_M(\mu)$. *Shreve* [1969] showed that topologic stream lengths often deviate markedly from a geometric series and, citing empirical studies, noted that natural networks also often violate Horton's law of stream lengths [*Shreve*, 1969, p. 407].

Table 1. Development of Test Samples

Sample	Threshold-Selection Criterion or Q Model Parameter
Topologically Random Samples	
0 and 0 mix	$Q = 1/2$
Type 1 Samples	
A and A mix	$R_B(\mu) \le \tilde{R}_B(\mu)$
B and B mix	$d(\mu) \ge \tilde{d}(\mu)$
C and C mix	$w(\mu) \ge \tilde{w}(\mu)$
D and D mix	$n\text{TS}(\mu) \ge n\widetilde{TS}(\mu)$
Type 2 Samples	
E and E mix	$Q = 0$
F and F mix	$Q = 1/6$
G and G mix	$Q = 1/3$
H and H mix	$Q = 2/3$
I and I mix	$Q = 5/6$

The median, for given network magnitude, μ, of each selection variable in the original topologically random sample of 10,000 networks is indicated by a tilde.

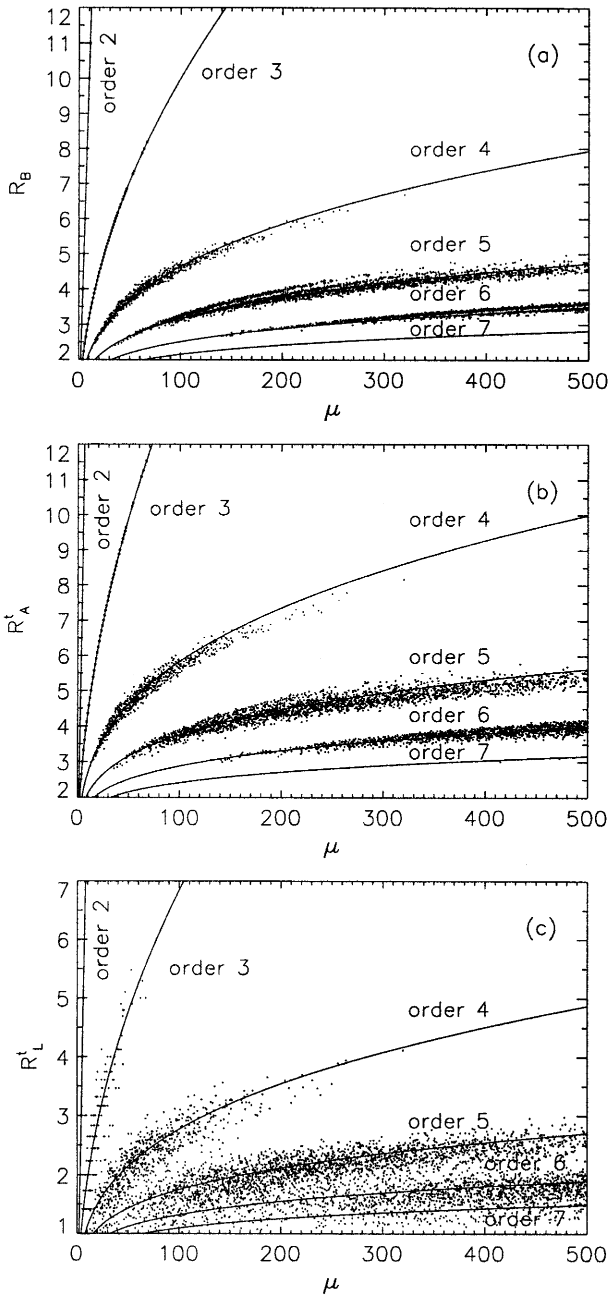

Figure 5. Topologic-analog Horton ratios plotted against network magnitude, μ, for topologically random network sample 0 mix: (a) R_B, (b) R_A^t, and (c) R_L^t. Curves are described by (4), (6), and (7) combined with (9), respectively. The most frequent order, $\Omega_M(\mu)$, is given by (5).

Holding network magnitude fixed in Figure 5, we obtain multimodal R_B, R_A^t, and R_L^t distributions. The R_B distribution for topologically random magnitude 50 sample 0 is shown in Figure 6a. This distribution has upper, central, and lower modes, corresponding to network orders 3, 4, and 5, respectively. Mode positions are approximated by the values of 7.07, 3.68, and 2.66, given by (4). The most frequent Strahler order, from (5), is 4. Sample 0 contains 2.0%, 87.2%, and 10.8% of networks of orders 3, 4, and 5, respectively, which is in close agreement with analytic predictions [*Shreve*, 1966, Table 2]. The minimum and maximum possible orders for magnitude 50 are 2 and 6 (see Figure 5a), but no networks of these orders appear in sample 0; their probability of occurrence in a topologically random sample of size 5000 is only 1.21×10^{-4} (computed from *Shreve* [1966, Table 2]).

The R_B distribution of sample A (Figure 6a) represents a truncated distribution of a topologically random sample because the selection variable used in constructing this type 1 sample is R_B itself. Type 1 samples B, C, and D, selected for variables d, w, and nTS, respectively, have R_B distributions differing little from sample 0 (Figure 6a). These results agree with those of *Kirchner* [1993], who studied test samples of type 1.

Some magnitude 50 test samples of type 2 have R_B distributions markedly different from sample 0 (Figure 6a). The most frequent order is 5 for samples E and F, 4 for G and H, and 3 for I. Order 5 networks are more frequent in samples E, F, and G ($Q < 1/2$) and less frequent in sample H ($Q > 1/2$) than in sample 0. No order 5 networks appear in sample I (highest Q value tested, $Q = 5/6$). Order 3 networks are less frequent in sample G ($Q < 1/2$) and more frequent in samples H and I ($Q > 1/2$) than in sample 0. No order 3 networks appear in samples E and F (lowest Q values tested, $Q = 0$ and

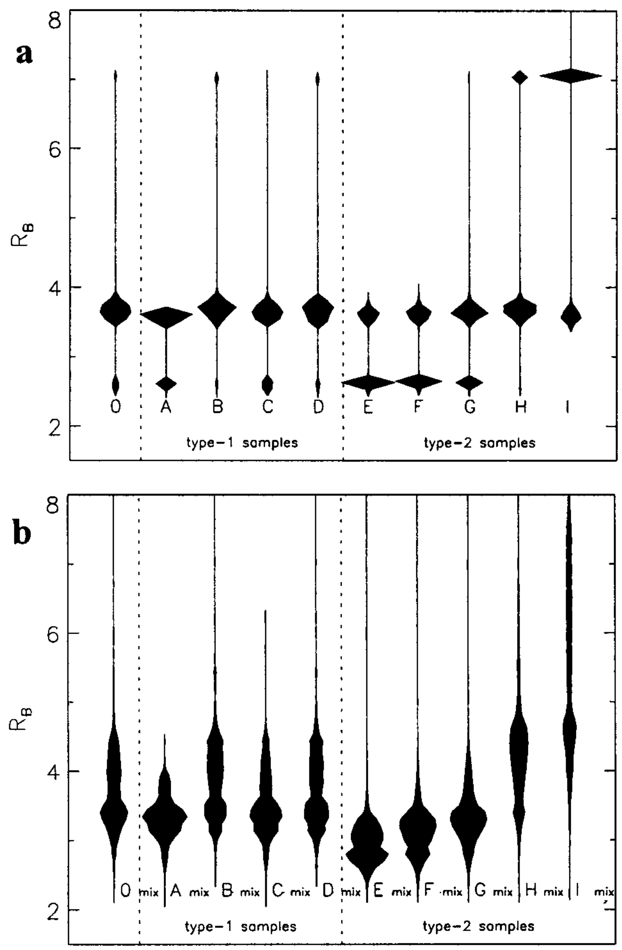

Figure 6. R_B distribution histograms for each test sample using a bin size of 0.1. For each sample the vertical reference axis extends from the minimum to maximum values observed, and the width about the axis is proportional to frequency. Where there is no thickness about the reference axis, there are no observed values. (a) Magnitude 50 test samples; (b) mixed-magnitude test samples.

$Q = 1/6$). Sample I ($Q = 5/6$) includes four networks of order 2 (having $R_B = 50$, which is above the plot axis range).

The positions of R_B, R_A^t, and R_L^t distribution modes depend on network magnitude ((4), (6), (7), and (9)); hence mixed-magnitude distributions do not have separated modes (Figure 6b). Increasing Q values in type 2 samples, from $Q = 0$ to $Q = 5/6$ (E mix through I mix), yields higher R_B, R_A^t, and R_L^t means and increased dispersion. This is shown for R_B in Figure 6b.

Results for the R_A^t and R_L^t distributions (not shown) are qualitatively similar to those for the R_B distributions shown in Figure 6. R_B, R_A^t, and R_L^t means and standard deviations for the mixed-magnitude test samples are given in Table 2. Figure 7 shows the R_B, R_A^t, and R_L^t means for the type 2 mixed-magnitude test samples, plotted against the Q parameter.

Horsfield et al. [1987] derived (10a) for the mean ratio between stream numbers of orders 1 and 2, $\bar{R}_{B_{1-2}}$, as a function of Q. For large magnitude, R_B approaches $\bar{R}_{B_{1-2}}$ (10b):

$$\bar{R}_{B_{1-2}} = 3 + \frac{Q}{1 - Q} \tag{10a}$$

$$R_B \approx \bar{R}_{B_{1-2}} \qquad \mu \text{ large} \tag{10b}$$

From (1) the ratio $Q/(1 - Q)$ in (10a) equals p_i/p_e.

The most frequent network order, $\Omega_M(\mu, Q)$, is given upon substitution of log ($\bar{R}_{B_{1-2}}$), obtained from (10a), for log (4) in (5). Smaller Q values yield larger $\Omega_M(\mu, Q)$ values. The position of the principal R_B distribution mode is estimated using (4) by substituting $\Omega_M(\mu, Q)$ for Ω. The R_B value obtained from (4) oscillates above and below the value given by (10). The amplitude of these oscillations decreases with μ and is larger for larger Q (see *Horsfield and Woldenberg* [1986] for an extensive discussion and related references). For the low range of magnitudes in our mixed-magnitude test samples, (10b) is not a good approximation, especially for large Q.

For $Q = 1/2$ (yielding topological randomness), the R_B value predicted by (10) is 4.0. For the maximum magnitude used, 500, the order which gives R_B closest to 4.0 is, from (5), $\Omega_M(500) = 5$. The R_B modal value for order 5, using (4), is 4.729. The principal modal value of R_B in a topologically random population oscillates about the value of 4.0.

The published observed values of the Horton ratios include

Table 2. Arithmetic Means and Standard Deviations of R_B, R_A^t, and R_L^t for Mixed-Magnitude Test Samples

	R_B		R_A^t		R_L^t	
Sample	Mean	s.d.	Mean	s.d.	Mean	s.d.
			Topologically Random Sample			
0 mix	3.767	0.605	4.448	0.940	2.005	0.531
			Type 1 Samples			
A mix*	3.448	0.356	4.029	0.549	1.792	0.357
B mix	3.881	0.612	4.586	0.994	2.162	0.540
C mix	3.614	0.488	4.249	0.701	1.857	0.399
D mix	3.796	0.614	4.498	1.000	2.026	0.563
			Type 2 Samples			
E mix	3.006	0.291	3.422	0.470	1.219	0.220
F mix	3.181	0.344	3.650	0.554	1.396	0.284
G mix	3.405	0.445	3.951	0.737	1.631	0.396
H mix	4.337	0.820	5.258	1.310	2.622	0.767
I mix	5.704	1.647	7.289	2.643	4.043	1.623

*The selection variable used in constructing sample A mix was R_B.

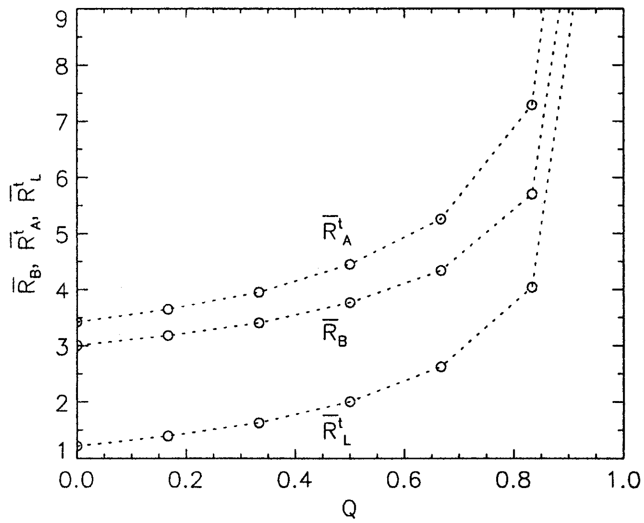

Figure 7. Mean values of the topologic-analog Horton ratios in mixed-magnitude test samples of type 2 and sample 0-mix, plotted against the respective Q parameter value: $Q = 0$ (sample E mix), $Q = 1/6$ (F mix), $Q = 1/3$ (G mix), $Q = 1/2$ (0 mix), $Q = 2/3$ (H mix), and $Q = 5/6$ (I mix). For $Q = 1$, only order 2 networks are created and, for the uniform mixture of magnitudes in [1, 500] used in our test samples, we have $\bar{R}_B = 252$, $\bar{R}_A^t = 507$, and $\bar{R}_L^t = 251$. Curves were constructed by linear interpolation.

networks of various magnitudes and from different environments. R_B is usually in the range of 3 to 5, with a modal value near 4; R_L is in the range of 1.5 to 3.5, with a modal value near 2; and R_A is in the range of 3 to 6, with a modal value near 4 [*Chorley*, 1957; *Smart*, 1972; *Abrahams*, 1984]. Shreve remarked that for a topologically random population of finite-magnitude networks, the R_B principal modal value and main range of variation coincide with the observed values [*Shreve*, 1966], and that if all network links had the same length and drainage area, R_L and R_A would coincide with their topologic analogs, R_L^t and R_A^t, which for a uniform distribution of TDCN of finite magnitude exhibit a range of variation similar to observations [*Shreve*, 1967, p. 184]. Shreve interpreted these results as supportive evidence for the RT model [*Shreve*, 1966, p. 36; *Shreve*, 1969, p. 414].

Kirchner [1993] showed that distributions of the Horton ratios in non–topologically random samples of type 1 do not deviate much from those in a random sample and concluded that these ratios have little sensitivity to topological distribution; hence the RT model prediction of their typical empirical values does not provide evidence in support of the RT model. We have shown that the Horton ratios are sensitive to the Q parameter in type 2 test samples. It is possible that they are sensitive also to other models of network growth.

Property 2: Correlations Among Horton Ratios and Between Stream Numbers and Stream Lengths

The topologic-analog Horton ratios depend strongly on magnitude, μ, and order, Ω (equations (4), (6), (7) and (9)). When μ is held fixed, the modal clusters corresponding to Ω (Figures 5 and 6a) are seen in plots of paired values of these ratios (Figures 8a and 9a for sample 0). Correlation coefficients between (R_B, R_L^t), (R_B, R_A^t), and (R_L^t, R_A^t) are determined largely by cluster positions and are high in all test

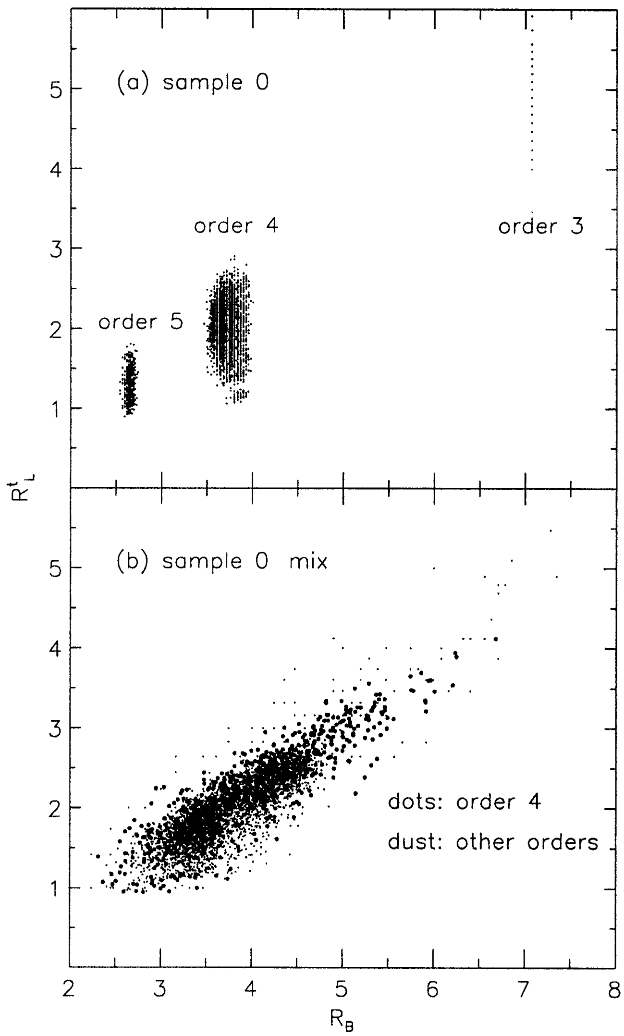

Figure 8. R^t_L plotted against R_B for each network in topologically random test samples (a) 0 ($\mu = 50$) and (b) 0 mix ($1 \le \mu \le 500$).

samples, especially for (R_B, R^t_A) (Table 3, magnitude 50 samples). When both μ and Ω are held constant, that is, within a cluster, (R_B, R^t_L) are uncorrelated, and the (R_B, R^t_A) and (R^t_L, R^t_A) correlations drop considerably (see Table 3, order 4 columns, magnitude-50 samples).

In mixed-magnitude samples, clusters overlap, and plots of paired values of R_B, R^t_A, and R^t_L show stratification by Ω, which is most marked in the case of (R_B, R^t_A) (Figure 9b). Correlation coefficients are high for all three pairs in all mixed-magnitude samples, especially for (R_B, R^t_A) (Table 3). Because of stratification by Ω, correlation coefficients are higher for Ω held fixed (see Table 3, (R_B, R^t_A) order 4 column, mixed-magnitude samples).

The mean ratios R^t_L/R_B, R^t_A/R_B, and R^t_L/R^t_A are higher for sets of networks of lower Ω ((4), (6) and (7)). Ratio R^t_A/R_B for fixed Ω is approximately equal to $2^{1/(\Omega-1)}$ (equations (4) and (6)) and tends to 1 for large Ω (Figure 9b). Because higher Q values tend to originate a larger proportion of networks of lower Ω, ratios R^t_L/R_B, R^t_A/R_B, and R^t_L/R^t_A increase with Q, that is, from sample E to I and from E mix to I mix (Table 3).

"Correlations between stream numbers and stream lengths," included by *Shreve* [1975] under property 2, refers to the prediction of the set of mean stream lengths, $\{\bar{L}_\omega\}$, from the set of stream numbers, $\{N_\omega\}$, via (8) for networks from topologically random samples, under the assumption of linearity between measured and topologic lengths (section 1). *Smart* [1968] showed that $\bar{L}^t_\omega$ from (8) provides an approximation to $\bar{L}_\omega/\bar{l}_i$, where $\bar{L}_\omega$ is the mean of observed lengths of streams of order ω, and $\bar{l}_i$ is the mean observed link length. Correlation coefficients between predicted and observed $\bar{L}_\omega$ ranged from 0.69 to 0.90 for various ω in basins analyzed by *Smart* [1968].

Expression (8) was derived for the RT model and is not expected to provide approximate $\bar{L}^t_\omega$ predictions for non–topologically random samples. A corresponding expression to (8) is not currently available for networks created by the Q model. To test the sensitivity of the relation between $\{\bar{L}^t_\omega\}$ and $\{N_\omega\}$, we compare the values observed in each test sample, $\bar{L}^t_{\omega_{obs}}$, to RT model predictions obtained from $\{N_\omega\}$ via (8), $\bar{L}^t_{\omega_{pred}}$, (Table 4).

In type 1 samples, (8) provides good predictions, which are comparable to those for sample 0 (Table 4). In type 2 samples the mean ratio between $\bar{L}^t_{\omega_{obs}}$ and $\bar{L}^t_{\omega_{pred}}$ varies with Q and ω, while correlation coefficients are high (0.737–0.916) for all test

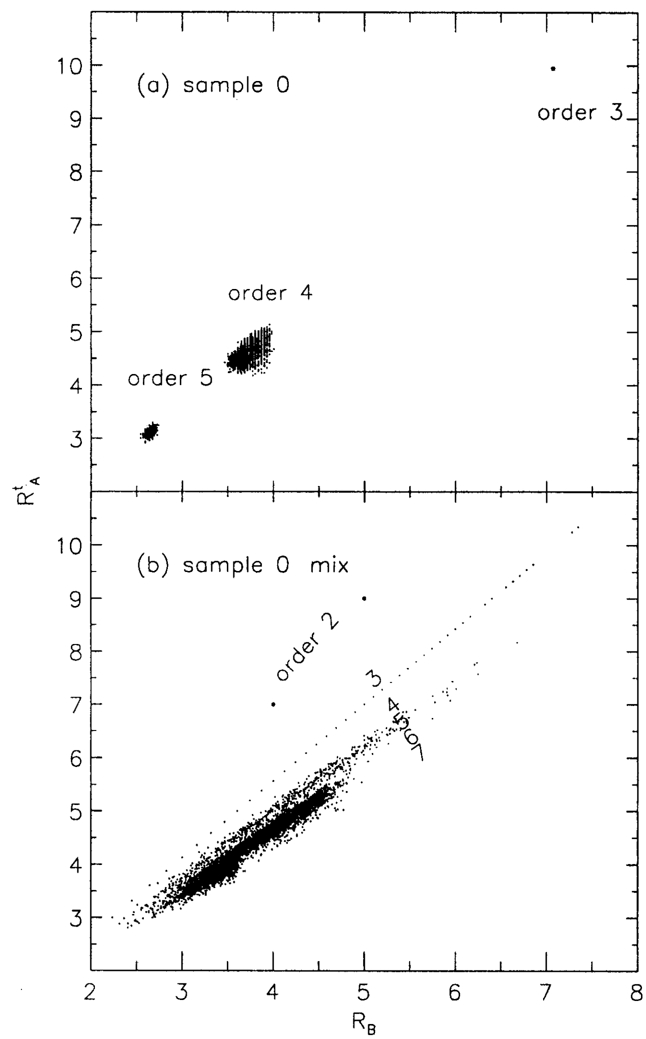

Figure 9. R^t_A plotted against R_B for each network in topologically random test samples (a) 0 ($\mu = 50$) and (b) 0 mix ($1 \le \mu \le 500$). There is marked stratification by order in sample 0 mix.

Table 3. Correlations, $r(\ ,\ )$, Between Topologic-Analog Horton Ratios in Test Samples and in Subsets Containing Only Those Networks of Order 4, and Mean Ratios Between These Variables

Magnitude-50 Samples	$r(R_B, R_L^t)$ All	$r(R_B, R_L^t)$ Order 4	$r(R_B, R_A^t)$ All	$r(R_B, R_A^t)$ Order 4	$r(R_L^t, R_A^t)$ All	$r(R_L^t, R_A^t)$ Order 4	R_L^t/R_B Mean	R_A^t/R_B Mean	R_L^t/R_A^t Mean
				Topologically Random Sample					
0	0.829	−0.073	0.990	0.661	0.801	−0.406	0.546	1.239	0.441
				Type 1 Samples					
A*	0.803	0.032	0.989	0.414	0.769	−0.261	0.547	1.169	0.484
B	0.850	−0.130	0.991	0.626	0.822	−0.493	0.579	1.243	0.467
C	0.706	−0.127	0.988	0.710	0.665	−0.392	0.521	1.233	0.423
D	0.844	−0.105	0.992	0.661	0.822	−0.398	0.542	1.248	0.435
				Type 2 Samples					
E	0.894	−0.220	0.997	0.791	0.881	−0.386	0.432	1.199	0.360
F	0.881	−0.154	0.996	0.762	0.863	−0.426	0.465	1.205	0.386
G	0.817	−0.087	0.992	0.718	0.786	−0.392	0.506	1.221	0.415
H	0.935	−0.057	0.997	0.560	0.924	−0.417	0.593	1.275	0.466
I	0.959	0.108	0.990	0.420	0.952	−0.201	0.697	1.354	0.513

Mixed-Magnitude Samples	$r(R_B, R_L^t)$ All	$r(R_B, R_L^t)$ Order 4	$r(R_B, R_A^t)$ All	$r(R_B, R_A^t)$ Order 4	$r(R_L^t, R_A^t)$ All	$r(R_L^t, R_A^t)$ Order 4	R_L^t/R_B Mean	R_A^t/R_B Mean	R_L^t/R_A^t Mean
				Topologically Random Sample					
0 mix	0.874	0.886	0.936	0.993	0.872	0.857	0.527	1.177	0.449
				Type 1 Samples					
A mix*	0.709	0.813	0.896	0.985	0.667	0.768	0.514	1.162	0.443
B mix	0.879	0.865	0.940	0.988	0.898	0.836	0.558	1.176	0.476
C mix	0.799	0.844	0.951	0.989	0.781	0.816	0.504	1.170	0.431
D mix	0.888	0.854	0.943	0.990	0.876	0.820	0.528	1.178	0.449
				Type 2 Samples					
E mix	0.745	0.732	0.904	0.995	0.870	0.700	0.405	1.137	0.356
F mix	0.778	0.796	0.908	0.992	0.858	0.762	0.437	1.145	0.382
G mix	0.834	0.827	0.922	0.994	0.871	0.797	0.476	1.157	0.412
H mix	0.900	0.904	0.936	0.991	0.880	0.869	0.598	1.206	0.497
I mix	0.954	0.912	0.957	0.976	0.923	0.855	0.694	1.267	0.550

*The selection variable used in constructing samples A and A mix was R_B.

samples and all ω. Correlations increase with network magnitude (not shown). Figure 10 illustrates the high correlation between $\bar{L}^t_{2_{\text{obs}}}$ and $\bar{L}^t_{2_{\text{pred}}}$ for samples 0 mix and E mix. The correlation for sample E mix is even higher than for sample 0 mix; however, the ratio $\bar{L}^t_{2_{\text{obs}}}/\bar{L}^t_{2_{\text{pred}}}$ is greater than 1 for sample E mix. This ratio is greater than 1 for $Q < 1/2$ and less than 1 for $Q > 1/2$ for order 2; approximately equal to 1 for all Q for order 3; and less than 1 for $Q < 1/2$ and greater than 1 for $Q > 1/2$ for higher orders (Table 4).

Property 3: Statistical Distributions of Second-Order Stream Lengths, Schumm Lengths, and Areas

The topologic analog of stream length is the number of links in the stream, and the analog of both Schumm length (total channel length in the basin) and drainage area is the total number of links in the network. In subbasins of order 2, the total number of links equals $2L_2^t + 1$, where L_2^t is the topologic length of the second-order stream.

Figure 11 shows the frequency distribution of L_2^t for all second-order streams in the networks of mixed-magnitude type 2 samples. Table 5 gives the mean and standard deviation for each test sample. For an infinite topologically random network, the frequency distribution of topologic lengths of streams of any given order ω is geometric [*Shreve*, 1969, equation 9b], and the expected topologic stream length for successive orders is given by a geometric series of ratio 2 [*Shreve*, 1967, p. 184; *Shreve*, 1969, equation 9c].

In type 1 samples the L_2^t frequency distribution differs little from RT model predictions (Table 5), the largest deviation being for sample D mix, selected for $n\text{TS}(\mu)$ higher than the median. In type 2 samples this distribution differs from RT model predictions and is not geometric (Figure 11, Table 5).

Property 4: Proportional Relationship Between Stream Frequency and the Square of Drainage Density

Stream frequency, F_s, and drainage density, D, are the average number of streams and average length of streams, respectively, per unit area in a drainage basin. *Melton* [1958, pp. 36–37] analyzed data for 156 basins of differing area, climate, relief, geology, and vegetation and found the ratio F_s/D^2 to be approximately constant:

$$\frac{F_s}{D^2} \approx 0.694 \tag{11}$$

Shreve [1967, pp. 184–185] showed that the topologic analog of F_s/D^2 is the ratio between the number of streams, S_s, and the number of links, $2\mu - 1$. The ratio $S_s/(2\mu - 1)$ is the reciprocal of the average number of links, or topologic length, per stream.

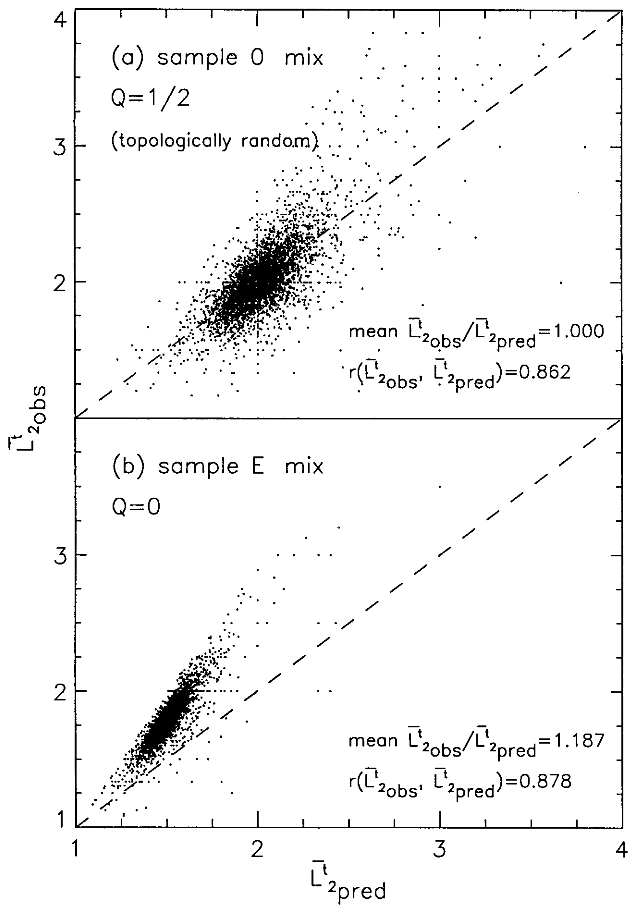

Figure 10. Average topologic length of second-order streams, $\bar{L}^t_{2_{obs}}$, plotted against RT model prediction, $\bar{L}^t_{2_{pred}}$, obtained from the stream number set using (8), for each network in samples (a) 0 mix and (b) E mix.

For large μ, expected stream numbers approximate a geometric series with ratio R_B, and the expected number of streams, S_s, is given by the sum of this series over orders 1 through Ω:

Table 4. Mean Ratio of Observed, $\bar{L}^t_{\omega_{obs}}$, to Predicted, $\bar{L}^t_{\omega_{pred}}$, Mean Topologic Stream Lengths and Corresponding Correlations, $r(\ \ ,\ \ )$, for Mixed-Magnitude Test Samples

	Mean ($\bar{L}^t_{\omega_{obs}}/\bar{L}^t_{\omega_{pred}}$)				$r(\bar{L}^t_{\omega_{obs}}, \bar{L}^t_{\omega_{pred}})$			
Sample	ω=2	ω=3	ω=4	ω=5	ω=2	ω=3	ω=4	ω=5
Topologically Random Sample								
0 mix	1.000	0.998	1.021	1.060	0.862	0.840	0.845	0.833
Type 1 Samples								
A mix	1.012	1.010	1.016	1.083	0.853	0.815	0.850	0.810
B mix	1.004	1.005	1.034	1.078	0.856	0.820	0.844	0.826
C mix	0.997	0.992	0.996	0.989	0.849	0.823	0.837	0.822
D mix	1.016	1.022	0.994	1.077	0.865	0.849	0.838	0.821
Type 2 Samples								
E mix	1.187	1.015	0.740	0.544	0.878	0.738	0.737	0.782
F mix	1.144	1.012	0.805	0.642	0.885	0.773	0.762	0.787
G mix	1.083	1.012	0.886	0.809	0.878	0.792	0.793	0.811
H mix	0.873	0.979	1.241	1.484	0.841	0.828	0.881	0.814
I mix	0.671	1.017	1.744	2.075	0.787	0.870	0.916	0.818

$\bar{L}^t_{\omega_{pred}}$ is the value predicted by the RT model, using (8).

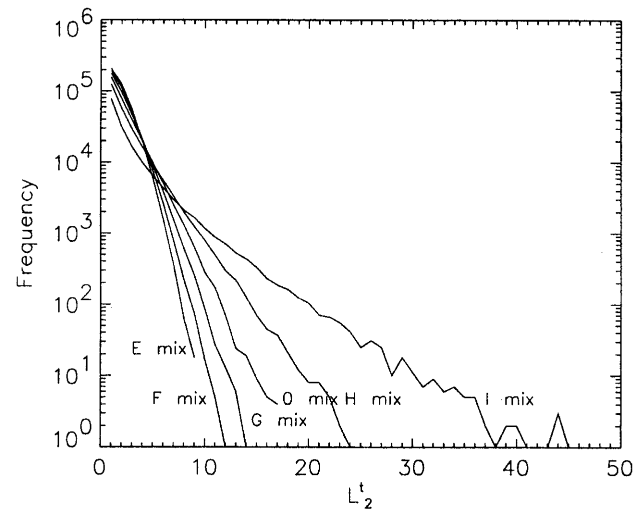

Figure 11. Frequency distribution of topologic length of second-order streams, L^t_2, for mixed-magnitude test samples of type 2.

$$S_s = \sum_{\omega=1}^{\Omega} R_B^{\Omega-\omega} = \frac{1 - R_B^{\Omega}}{1 - R_B} \tag{12}$$

For large-μ, topologically random networks, we have $R_B \approx 4$, and substituting (5) for Ω in (12) yields $S_s \approx 4\mu/3$ and $S_s/(2\mu - 1) \approx 2/3$ [*Shreve*, 1967, p. 185]. This value compares well with (11).

For large-magnitude networks generated by the Q model, substituting log (R_B) for log (4) in (5), we obtain from (12)

$$\frac{S_s}{2\mu - 1} \approx \frac{R_B}{2(R_B - 1)} \tag{13}$$

Substituting (10) into (13) yields

$$\frac{S_s}{2\mu - 1} \approx \frac{3 - 2Q}{4 - 2Q} \tag{14}$$

Table 6 gives the mean and standard deviation of $S_s/(2\mu - 1)$ for each test sample. Mean values for type 1 samples are close to those of sample 0. Mean values for type 2 samples are in

Table 5. Mean and Standard Deviation of $\bar{L}^t_2$ for Mixed-Magnitude Test Samples

	$\bar{L}^t_2$	
Sample	Mean	s.d.
Topologically Random Sample		
0 mix	2.003	1.411
Type 1 Samples		
A mix	1.996	1.298
B mix	1.999	1.430
C mix	1.991	1.301
D mix	2.139	1.427
Type 2 Samples		
E mix	1.792	0.999
F mix	1.838	1.087
G mix	1.905	1.217
H mix	2.177	1.790
I mix	2.802	2.386

Table 6. Mean and Standard Deviation of $S_s/(2\mu - 1)$ for Mixed-Magnitude Test Samples

	$S_s/(2\mu - 1)$	
Sample	Mean	s.d.
Topologically Random Sample		
0 mix	0.672	0.022
Type 1 Samples		
A mix	0.672	0.022
B mix	0.670	0.022
C mix	0.672	0.022
D mix	0.667	0.020
Type 2 Samples		
E mix	0.751	0.020
F mix	0.729	0.021
G mix	0.703	0.022
H mix	0.633	0.022
I mix	0.582	0.022

approximate agreement with (14). For the purpose of illustration the $S_s/(2\mu - 1)$ values in test samples 0 mix and E mix are plotted against μ in Figure 12.

Property 5: Variation of Mainstream Length With Basin Area

Hack [1957] found an exponential relation between mainstream length, L, and basin area, A:

$$L = \beta A^{\alpha} \tag{15}$$

Hack [1957] and *Gray* [1961] found the exponent α to vary with geographic location, taking values larger than 0.5. *Mueller* [1973] collected data for several thousand basins of all sizes in various parts of the world, and found that α changed abruptly from 0.6 for basins of area smaller than 8000 square miles (20,720 km^2), to 0.5 for basins between 8000 and 100,000 square miles (20,720–259,000 km^2), and to 0.466 for basins larger than 100,000 square miles (259,000 km^2).

The computed value of α (15) is subject to uncertainty. *Robert and Roy* [1990] [see also *Beer*, 1991; *Robert and Roy*, 1991] presented evidence of influence of cartographic scale on the estimated α. *Montgomery and Dietrich* [1992] replaced stream lengths by basin lengths, measured along the main valley axis to the basin divide, thereby obtaining $\alpha = 0.5$. These authors suggested that α depends on the headward extent of the mainstream depicted on maps of different scale as well as on downstream variations in channel sinuosity and drainage density.

The topologic analogs of L and A are network diameter, d, and total number of links, $2\mu - 1$, respectively. However, when data from basins of different drainage densities are lumped together, there is not a direct correspondence between L and A and their topologic analogs. Despite this limitation, (15) has been used to test the RT model using the topologic analogs.

Mesa and Gupta [1987, equation (16)] derived the approximate expression (16) for α for the RT model, under the assumption of independent, identically distributed (i.i.d.) exponential link lengths:

$$\alpha(\mu) = \frac{1}{2}\,\frac{\pi + (\pi/\mu)^{1/2}}{\pi - (1/\mu)} \tag{16}$$

Expression (16) gives that α, for the RT model, decreases continuously from about 0.6 for small magnitudes, μ, to the asymptotic value of 0.5 for large μ. Similar results had been obtained by Monte Carlo simulations by *Shreve* [1970] and *Werner and Smart* [1973].

Shreve [1974, pp. 1175–1176] showed that the observed deviations from RT model predictions for large basin areas may be accounted for by channel sinuosity (see also *Smart and Surkan* [1967]). *Mesa and Gupta* [1987] remark that the abrupt changes of α with basin size cannot be accounted for by the RT model. In our view the above uncertainties concerning the computation of α from observed data preclude any definite conclusions regarding the validity of the RT model. Moreover, the above computed values of α may include basins subject to geologic controls.

Figure 13b shows a log-log plot of diameter, d, versus the number of links, $2\mu - 1$, for topologically random sample 0 mix. The values of d in sample 0 mix lie within a narrow region inside the zone of possible values, and the variance does not decrease with magnitude, as shown analytically by *Mesa* [1986], cited by *Mesa and Gupta* [1987, equation (27)], so that $d(\mu)$ does not tend to almost-sure values at large μ. Figures 13a and 13c illustrate the influence of Q on the $d(\mu)$ distribution. The

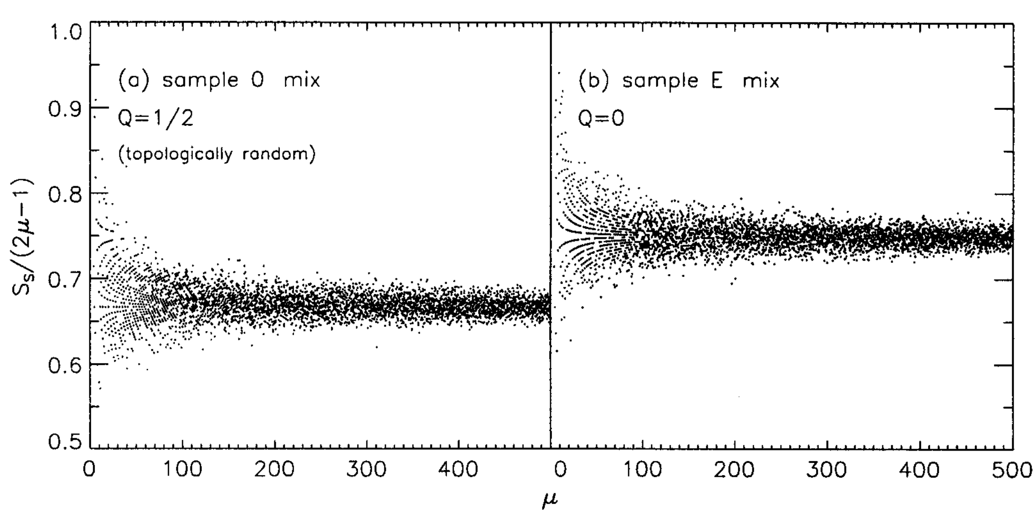

Figure 12. Number of streams, S_s, divided by number of links, $2\mu - 1$, plotted against magnitude, μ, for test samples (a) 0 mix and (b) E mix. For large μ, $S_s/(2\mu - 1)$ is approximated by (14).

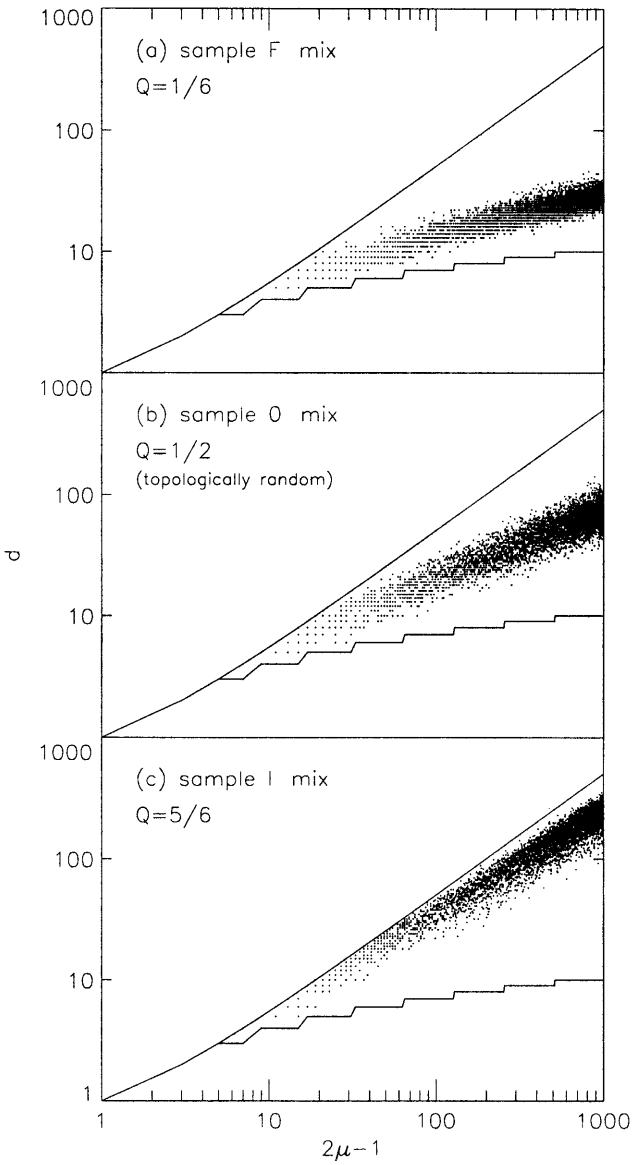

Figure 13. Diameter, d, plotted against number of links, $2\mu - 1$, for samples (a) F mix, (b) 0 mix, and (c) I mix. Solid lines delineate zone of possible values.

$d(\mu)$ distributions of samples F mix and I mix, pictured, do not overlap for μ larger than about 70.

Figure 14 shows similar plots for test samples of type 1. Deviation from the RT model (Figure 13b) is small in all type 1 samples, excepting the sample obtained by selection of $d(\mu)$ larger than the median, $\tilde{d}(\mu)$. The reasons for the small deviation are addressed in section 5, the conclusions.

The value of α varies with μ in all samples. Table 7 gives the overall α and β values. Both α and β vary markedly with Q; α varies from less than 0.3 for $Q = 0$ (sample E mix) to 1.0 for $Q = 1$ (order 2 networks, represented by the upper boundary of the zone of possible values of $d(\mu)$ in Figures 13 and 14).

Property 6: Variation of Basin Distance-Weighted Area With Basin Area

Langbein et al. [1947] studied the relation between basin area, A, and the basin's "distance-weighted area," P, defined by

$$P = \sum_k l_k a_k \tag{17}$$

where a_k is a portion of the basin area (a "partial area"), and l_k is the channel distance from the partial area to the outlet. P

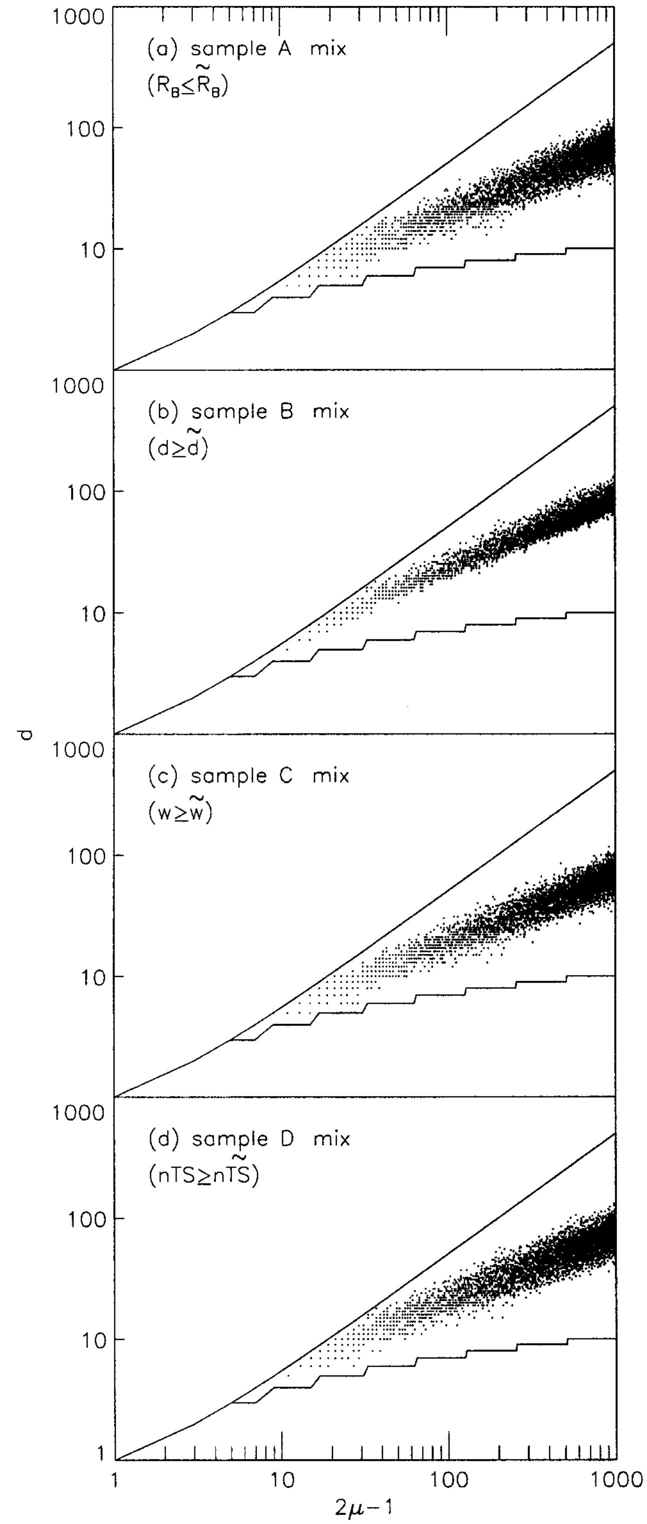

Figure 14. Diameter, d, plotted against number of links, $2\mu - 1$, for samples of type 1: (a) A mix, (b) B mix, (c) C mix, and (d) D mix. The selection variable used to create sample B mix was d itself. Solid lines delineate zone of possible values.

has dimension of length cubed. Using data for 340 drainage basins in the northeastern United States, *Langbein et al.* [1947] found the relation

$$P = 0.90A^{1.56} \tag{18}$$

where P and A are expressed in miles and square miles, respectively.

The topologic path length of a link is the number of links composing the path that connects the link of interest to the root link, including the link of interest itself. The total topologic path length, p, of a network is the sum of topologic path lengths of all links composing the network. If we choose the a_k values in (17) to be the individual link areas, and under the assumption that a_k and l_k are uncorrelated, then p is the topologic analog of P [*Werner and Smart*, 1973, p. 292].

Werner and Smart [1973] found the following relation for a computer-generated sample of 200 topologically random networks with magnitudes, μ, randomly selected from the range [20–200]:

$$p = 1.0(2\mu - 1)^{1.53} \tag{19}$$

Mesa and Gupta [1987, equation (25)] derived for the RT model, under the assumption of i.i.d. exponentially distributed link lengths, the approximate expression for p at large μ:

$$p \approx 2\pi^{1/2}\mu^{1.5} \qquad \mu \text{ large} \tag{20}$$

Figure 15 shows log-log plots of p versus $2\mu - 1$, for sample 0 mix, and type 2 samples F mix and I mix. The p values in sample 0 mix (topologically random) lie within a narrow plot region within the zone of possible values (Figure 15b). Table 8 gives the overall exponent (α') and coefficient (β') in (19) for each test sample. Variation of α' and β' is slight for type 1 samples but marked for type 2 samples. Both α' and β' vary with Q, and α' decreases from 1.242 for $Q = 0$ (sample E mix) to 1.803 for $Q = 5/6$ (sample I mix).

Table 7. Values of Exponent α and Coefficient β in $d = \beta(2\mu - 1)^{\alpha}$ for Mixed-Magnitude Test Samples, Obtained by Least-Squares Linear Regression of log (d) Over log ($2\mu - 1$), and Values of the Standard Deviation About the Regression Line

Sample	α	β	s.d.
	Topologically Random Sample		
0 mix	0.560	1.538	0.207
	Type 1 Samples		
A mix	0.556	1.487	0.195
B mix*	0.581	1.560	0.138
C mix	0.551	1.488	0.181
D mix	0.557	1.599	0.206
	Type 2 Samples		
E mix	0.272	3.227	0.110
F mix	0.355	2.595	0.148
G mix	0.451	2.016	0.183
H mix	0.689	1.100	0.217
I mix	0.832	0.772	0.191

Standard deviation values in units of log (d).
*The selection variable used in constructing sample B mix was d itself.

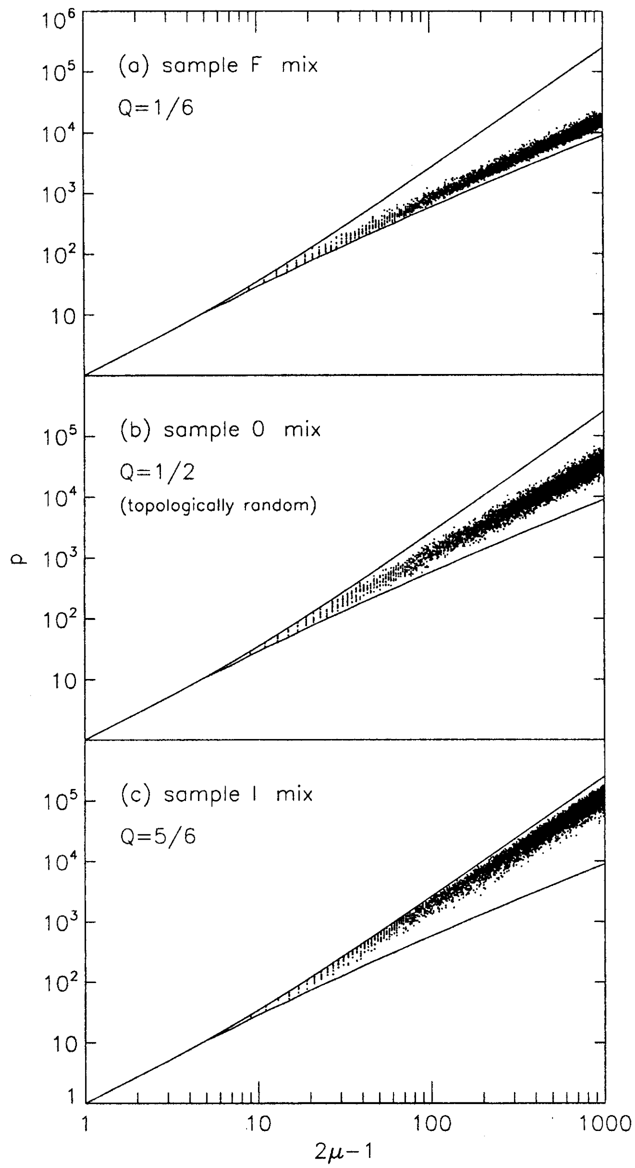

Figure 15. Total topologic path length, p, plotted against number of links, $2\mu - 1$, for samples (a) F mix, (b) 0 mix, and (c) I mix. Solid lines delineate zone of possible values.

Property 7: Relation Between Distance From the Basin Outlet to Centroid and Mainstream Length

Gray [1961] studied the relation between the channel distance from the outlet to the center of gravity (or "centroid") of the network, L_{ca}, and the mainstream length, L. L_{ca} is given by

$$L_{ca} = P/A \tag{21}$$

where P is defined by (17). *Gray* [1961] found the relation

$$L_{ca} = 0.5L \tag{22}$$

The topologic analogs of L and L_{ca} are d and the mean topologic path length, $\bar{p} = p/(2\mu - 1)$, respectively. *Werner and Smart* [1973, equation 34] obtained the following regression relation for a computer-generated sample of 200 topologically random networks with magnitudes μ randomly selected from the range [20–200]:

Table 8. Values of Exponent α' and Coefficient β' in $p = \beta'(2\mu - 1)^{\alpha'}$ for Mixed-Magnitude Test Samples, Obtained by Least-Squares Linear Regression of log (p) Over log ($2\mu - 1$), and Values of the Standard Deviation About the Regression Line

Sample	α'	β'	s.d.
	Topologically Random Sample		
0 mix	1.521	1.038	0.207
	Type 1 Samples		
A mix	1.515	1.021	0.201
B mix	1.546	1.011	0.161
C mix	1.510	1.023	0.192
D mix	1.520	1.063	0.212
	Type 2 Samples		
E mix	1.242	2.255	0.072
F mix	1.316	1.842	0.128
G mix	1.410	1.410	0.174
H mix	1.652	0.721	0.226
I mix	1.803	0.476	0.199

Standard deviation values in units of log (p).

$$\bar{p} = 0.50d + 1.3 \tag{23}$$

Both $\bar{p}$ and d increase with μ, and for large μ, (23) is approximated by $\bar{p} = 0.5d$, the topologic equivalent of (22). *Mesa and Gupta* [1987, equation (26)] derived for the RT model, at large μ, assuming i.i.d. exponentially distributed link lengths,

$$\bar{p} = 0.50d \qquad \mu \text{ large} \tag{24}$$

Because there is uncertainty in both d and $\bar{p}$ for given μ, it is not appropriate to perform regression of $\bar{p}$ against d. Regression implicitly assumes there to be uncertainty only in the dependent variable [e.g., *Hirsch and Gilroy*, 1984]. Instead, we computed the mean and standard deviation of $\bar{p}/d$ for each test sample (Table 9). Also given in Table 9 are the mean and standard deviation of $\bar{p}^*/d$, where $\bar{p}^*$ denotes the mean value of p^*, the "total modified topologic length," introduced here. The modified topologic length is here defined as the number of links forming the path that connects the link of interest and the root link, excluding the link of interest. The standard definition of topologic length includes the link of interest. We have

Table 9. Mean and Standard Deviation of $\bar{p}/d$ and $\bar{p}^*/d$ for Mixed-Magnitude Test Samples

	$\bar{p}/d$		$\bar{p}^*/d$	
Sample	Mean	s.d.	Mean	s.d.
		Topologically Random Sample		
0 mix	0.539	0.057	0.509	0.052
		Type 1 Samples		
A mix	0.539	0.058	0.509	0.053
B mix	0.532	0.055	0.507	0.052
C mix	0.543	0.059	0.512	0.055
D mix	0.536	0.055	0.508	0.051
		Type 2 Samples		
E mix	0.587	0.044	0.521	0.038
F mix	0.566	0.051	0.514	0.045
G mix	0.549	0.055	0.511	0.048
H mix	0.529	0.054	0.508	0.050
I mix	0.520	0.045	0.506	0.041

$$p^* = p - (2\mu - 1) \tag{25}$$

$$\bar{p}^* = \bar{p} - 1 \tag{26}$$

The reason for using p^* instead of p is that the ratio $\bar{p}^*/d$ shows dependency on μ only for very low μ values. Figure 16 depicts $\bar{p}^*/d$ plotted against μ for test samples F mix, 0 mix, and I mix. The mean $\bar{p}^*/d$ increases with Q, as does the standard deviation (Table 9). This increase is slight except for very low Q values.

For given d, variable $\bar{p}^*$ is determined by the network's shape. Networks having more links at higher than lower topologic distances from the root, said to be "top-heavy," have high $\bar{p}^*$ values. "Bottom-heavy" networks have more links at short topologic distances from the root and have low $\bar{p}^*$ values. Networks produced with low Q values tend to be more compact and with lower bifurcation ratios. Because the maximum number of links at a given topologic distance from the root increases with distance, these compact networks tend to have more links at topologic distances higher than half the network's diameter and hence are top heavy. Networks produced with high Q values do not have a tendency to be either top- or bottom-heavy. For $Q = 1$, which produces exclusively second-order networks, $\bar{p}^*/d$ tends to 0.5 for μ large.

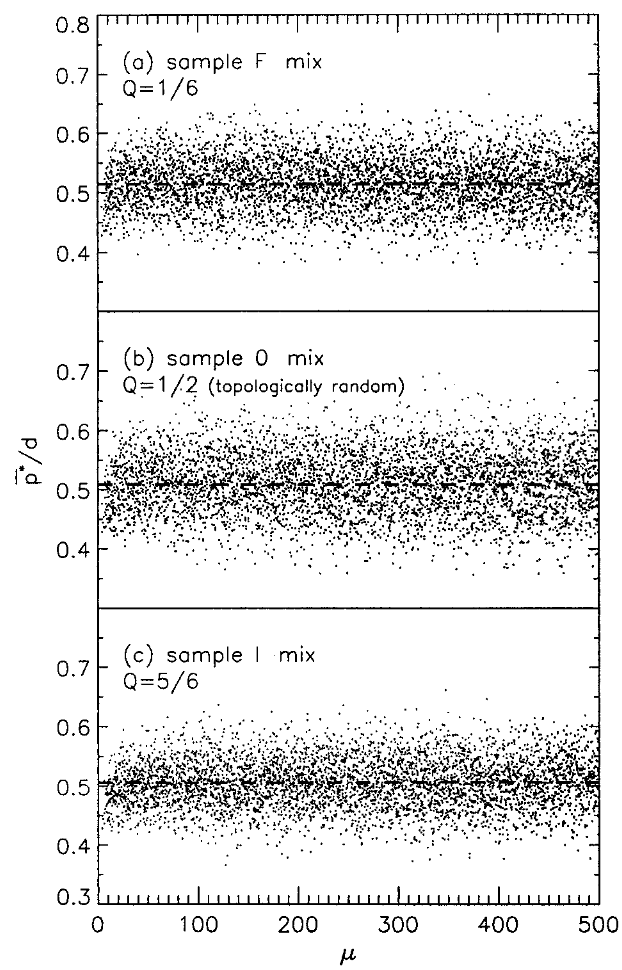

Figure 16. Ratio of mean modified total topologic length to diameter, $\bar{p}^*/d$, plotted against magnitude, μ, for each network in samples (a) F mix, (b) 0 mix, and (c) I mix. Dashed line represents the mean $\bar{p}^*/d$ value.

5. Conclusions

The sensitivity of channel network planform laws (properties 1–7) to nonuniform TDCN frequencies was tested by computing the coefficients of each topologic-analog law for non–topologically random test samples. Two types of test sample were used (section 3). Samples of type 1 were obtained by selection of networks from an original topologically random sample on the basis of threshold values of a specific topologic variable, located at the sample median. Samples of type 2 were obtained by constructing each network using the Q model of stochastic topologic growth (section 2), where the probabilities of tributary development on an exterior link, and on an interior link, are allowed to vary.

Sensitivity of the topologic-analog laws is different for type 1 and type 2 test samples (section 4). Sensitivity of all laws to type 1 samples (A through D and A mix through D mix) is limited. This is because the cutoff value for each selection variable was placed at the sample median. Given the sharply peaked distributions of the selection variables, a cutoff value equal to the sample median will not succeed in sampling away from the narrow distribution peak. The effects on topologic variables other than the selection variable depend on the correlations between variables and are small for cutoff values placed at the sample median. Cutoff values corresponding to the upper or lower third, fourth, fifth, or any decile would require larger sample sizes than those used here. Cutoff values located farther from the median would necessarily result in larger deviations from RT model predictions.

Parameter values of the Q model are well reflected in departures from RT model predictions for all topologic-analog laws. Only Gray's relation (property 7) shows very limited sensitivity to Q except for very low Q values. It is possible that properties 1–6 are sensitive to other models of network growth as well.

Correlations between (R_B, R_L^t), (R_B, R_A^t), and (R_L^t, R_A^t) are largely determined by network magnitude and order. When both these variables are held fixed, correlations drop considerably and are close to zero for (R_B, R_L^t). Given that observed correlations among these ratios [*Smart*, 1968] have not been studied conditionally for μ and Ω, the correlation values found do not establish well-defined geomorphologic laws. Conditional analysis of observed correlations and comparison with RT model predictions may constitute useful future work, especially since high correlations between R_B and R_L^t in natural channel networks may be a result of the space-filling requirements of these networks. *Tarboton et al*. [1988] and *La Barbera and Rosso* [1989] have proposed that the ratio $\log (R_B)/\log (R_L^t)$ provides an estimate of a network's fractal dimension, in the vicinity of 2.

Given the sensitivity of channel network planform laws to at least one class of network growth models, represented by the Q parameter, we conclude that the approximate agreement of these laws with the predictions of the RT model may be a distinctive property of channel networks rather than an inescapable result. The approximate agreement with the RT model remains unexplained. In attempting an explanation we caution that all models used here to create non–topologically random network samples are purely topological. Space-filling and geometric properties of natural channel networks were ignored and could lead to different results.

With the current availability of digital elevation maps and satellite imagery, from which approximate representations of channel networks may be obtained, the ability to interpret network morphology would be most useful for the inference of geophysical processes and geologic properties at the planetary scale. It is possible that the sensitivity of channel network planform laws to network growth models may render geographic variations in the coefficients of these laws useful for inference of different physical processes of network growth. Before such connections between process and form can be established, it is first necessary to assess the influence of space filling and geometry on network topology and on the sensitivity of geomorphologic laws to network growth models. The sensitivity of these laws to network growth models operating under spatial constraints is the subject of current research by the authors.

Notation

A	basin area.
d	diameter.
D	drainage density.
F	link frequency.
F_s	stream frequency.
G_B, G_L^t, G_A^t	geometric-mean bifurcation ratio, and topologic-analog length and area ratios.
L	mainstream length.
L_{ca}	channel distance from outlet to network center of gravity.
L_ω	length of a stream of order ω.
L_ω^t	topologic length (number of links) of a stream of order ω.
N_ω	number of streams of order ω.
nTS	number of tributary-source links in a network.
P	distance-weighted basin area.
p, p^*	standard and modified total topologic path length in a network.
Q	parameter of the "Q model."
R_B, R_L, R_A	arithmetic-mean bifurcation, length, and area ratios.
R_L^t, R_A^t	topologic analogs of R_L and R_A.
S_s	number of Strahler streams in a network.
w	network width.
μ	magnitude of a link or network.
Ω	Strahler order of a network.
$\Omega_M(\mu)$	most frequent Strahler order in TDCN of magnitude μ.
ω	Strahler order of a link or a Strahler stream.

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